ДИСКУРС. 2023. Т. 9, № 2. С. 103–110
DISCOURSE. 2023, vol. 9, no. 2, pp. 103–110

Языкознание
Linguistics

Original paper
УДК 81–116
http://doi.org/10.32603/2412-8562-2023-9-2-103-110

R-linguistics. The Principle of Linguistic Duality

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Introduction. R-linguistics uses the axiomatic method in its approach to modeling the world and language [1]. Axioms define the categories of language, their properties and ways of formation. On this basis, when processing the accumulated data in the form of relations, it is possible to form systems of categories and determine the verbs connecting these categories. A reasonable question arises: if categories (to be such) must satisfy certain axioms, does this requirement apply to verbs as well? The purpose of this article is to deal with this issue.

Methodology and sources. The results of the previous parts of the series are used as research tools. The axioms and methods of verb categorization formulated earlier are used to develop the necessary mathematical representations of the verb axiomatics.

Results and discussion. The article introduces the axiom for the verbs of the language (the axiom of the verbal cross) and shows that this axiom is related to the categorization axiom (the correctness axiom). Although this connection is not identical, it becomes such with the additional use of the axiom of extensiveness. This relationship made it possible to formulate the fundamental principle of linguistics – the principle of duality.

Conclusion. The axiomatics of verbs and categories of a language turns out to be connected, and one determines the other, so that by and large it does not matter which of them is the source of the model of the world and the appearance of language. This unity is formulated as the principle of duality in linguistics. The manifestations of the principle of duality in the language are diverse, but this article focuses on the discussion of the appearance in the language of verbs-processes and adverbs. This transition is considered, starting with obtaining the initial data about the world, to the emergence of ideas about processes and adverbs as a feature system of verbs.

Keywords: R-linguistics, categorization, verbal cross, duality principle

For citation: Polyakov, O.M. (2023), "R-linguistics. The Principle of Linguistic Duality", *DISCOURSE*, vol. 9, no. 2, pp. 103–110. DOI: 10.32603/2412-8562-2023-9-2-103-110

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Оригинальная статья

R-лингвистика. Принцип двойственности

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Введение. R-лингвистика в своем подходе к моделированию мира и языку использует аксиоматический метод. Аксиомы определяют категории языка, их свойства и способы формирования. На этой основе при обработке накопленных данных, оформленных в виде отношений, можно сформировать системы категорий и определить глаголы, соединяющие эти категории. Возникает резонный вопрос: если категории (чтобы быть таковыми) должны удовлетворять определенным аксиомам, относится ли это требование и к глаголам? Цель данной статьи – разобраться с указанным вопросом.

Методология и источники. Результаты предыдущих частей серии используются в качестве инструментов исследования. Сформулированные ранее аксиомы и методы глагольной категоризации применяются для разработки необходимых математических представлений глагольной аксиоматики.

Результаты и обсуждение. В статье введена аксиома для глаголов языка (аксиома глагольного креста) и показано, что эта аксиома связана с аксиомой категоризации (аксиомой корректности). Эта связь хотя и не тождественна, но становится таковой при дополнительном использовании аксиомы экстенсивности. Указанная взаимосвязь позволила сформулировать фундаментальный принцип лингвистики – принцип двойственности.

Заключение. Аксиоматика глаголов и категорий языка оказывается связанной, и одно определяет другое, так что по большому счету не важно, что из них является источником модели мира и появления языка. Это единство сформулировано как принцип двойственности в лингвистике. Проявления принципа двойственности в языке многообразны, но в данной статье внимание сосредоточено на обсуждении появления в языке глаголов-процессов и наречий. Этот переход рассмотрен начиная с получения исходных данных о мире до возникновения представлений о процессах и наречиях как признаковой системы глаголов.

Ключевые слова: R-лингвистика, категоризация, глагольный крест, принцип двойственности

Для цитирования: Поляков О. М. R-лингвистика. Принцип двойственности // ДИСКУРС. 2023. Т. 9, № 2. С. 103–110. DOI: 10.32603/2412-8562-2023-9-2-103-110.

Introduction. R-linguistics uses the axiomatic method in its approach to modeling the world and language [1]. Axioms define the categories of language, their properties and ways of formation. On this basis, when processing the accumulated data in the form of relations, it is possible to form systems of categories and determine the verbs connecting these categories. A reasonable question arises: if categories (to be such) must satisfy certain axioms, does this requirement apply to verbs as well? The purpose of this article is to deal with this issue.

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Results and discussion.

Axiomatics of verbs. The mathematical relations on which the verb categorization is built [1] represent the Galois correspondence. The Galois correspondence in the middle of the last century was generalized by the American mathematician Garrett Birkhoff [2]. In 1953, Jurgen Schmidt [3] formulated Axiom A3, which we will call the verbal cross:

$$A3 \qquad X \subseteq Y^{+} \text{ if and only if } Y \subseteq X^{*},$$

where * is a mapping of subsets of the universe U into subsets of the universe V, and + is a mapping of subsets of the universe V into subsets of the universe U. Together with the axiom A1 (the axiom of extensiveness).

$$A1' \qquad X \subseteq X^{*+} \text{ and } Y \subseteq Y^{+*},$$

which now acts on the universes U and V, we just get the definition of the Galois correspondence. However, so far there is no guarantee that the mappings *+ or +* are closure operators.

Proposition 1. For the verbs Δ and ∇ from [1], axiom A3 holds.

Proof. From Proposition 24 [1] we obtain the following: if $X \subseteq Y^{\nabla}$, then $Y^{\nabla\Delta} \subseteq X^{\Delta}$. According to Proposition 25 [1] $Y \subseteq Y^{\nabla\Delta}$, therefore $Y \subseteq X^{\Delta}$. Similarly, you can justify the validity of A3 in the opposite direction.

Proposition 2. The fulfillment of axioms A1 and A3 implies the fulfillment of the axiom A2 on both universes.

Proof. Now let Axioms A1 and A3 hold. If $X_1 \subseteq X_2^{*+}$ is true, then $X_2^{*} \subseteq X_1^{*}$ is true by A3. By virtue of A1, it is true that $X_1^{*} \subseteq X_1^{*+*}$, and therefore $X_2^{*} \subseteq X_1^{*+*}$. Hence, by virtue of A3, $X_1^{*+} \subseteq X_2^{*+}$. But this is exactly Axiom A2. Similarly, the validity of A2 can be substantiated for the universe V.

So, on the universes U and V, when the axioms A1 and A3 are fulfilled, linguistic spaces are formed. By Lemma 3 [1], the intersection of categories in these spaces is also a category. Actually, this is how we defined the verbs Δ and ∇.

An analogue of Proposition 29 [1] holds for *, +.

Proposition 3. Linguistic spaces with respect to the operators * + and + (+ * and *) coincide.

Proof. Indeed, by virtue of A1 (on V), for any $X \subseteq U$ we have $X^{*} \subseteq X^{*+*}$. By virtue of axioms A1 and C1, we have $X \subseteq X^{*+*+}$. Then from A3 we obtain $X^{*+*} \subseteq X^{*}$, which means that $X^{*} = X^{*+*}$. Likewise, for another universe. Thus, the mapping of any set X(Y) by the mapping * (+) immediately leads us to a closed set on U(V).

Since the self-evident axiom A1 is present in both cases, then in fact the meaning of the equivalence of the pairs of axioms A1, A2 and A1, A3 is concluded in the ratio of the axioms A2 and A3 (when it comes to binary verbs).

Remark 4. Let's consider what is the meaning of axiom A3 (axiom of the verbal cross). For example, suppose the universe U consists of many different fruits of plants, and the universe V consists of many different products that are made from fruits. So, let Y be some arbitrary set of products, for example, juice, pie, jam, etc. Then Y^{+} is the set of all fruits from which all these products can be made. In other words, from each fruit included in Y^{+}, you can make all the products included in the set of examples of Y. If you choose only part of the fruits X from Y^{+}, then of course you can make all the products from Y, but it may be possible to make some more products that could not be made because of some fruits included in Y^{+}, but not included in X. If denote by X^{*}

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the set of all products that can be made from X, then obviously with necessity $Y \subseteq X^*$. Similarly, one can show that $X \subseteq Y^+$ follows from $Y \subseteq X^*$. In other words, a decrease in the composition of objects at one universe leads to their increase at another, and vice versa. A consequence (manifestation) of this cross is Proposition 24, Lemma 32, and Theorem 33 from [1]. On the other hand, the correctness axiom A2 says that no subset of examples of a category allows one to go beyond this category.

As another example of the manifestation of the axiom of the verbal cross, let us consider the sentences: “добытчик добывает добычу”, “завариватель заваривает заварку”, “прихожанин приходит в приход”, “рыбаки рыбачат”. Usually, people in such cases talk about tautological statements, although this is not so. The reason for the “unacceptability” of this kind of expression is the violation of the axiom of the verbal cross, since they use universes as a category and co-category for the corresponding verb. This is easy to see if we reduce the level of community of one of the categories participating in the proposal, for example, “Василий Иванович приходит в приход”, “прихожанин приходит в часовню”, etc. We seem to restore the action of the verbal cross, which is felt by us as a harmonization of the text.

Verbs can be related in the relation “more-less” [4]. Recall that the verb V1 is less than the verb V2 if the linguistic spaces according to V1 belong to the linguistic spaces according to V2. For example, ПРИСТРАИВАТЬ ≤ СТРОИТЬ, ПРЕПОДАВАТЬ ≤ ОБУЧАТЬ, РЫБАЧИТЬ ≤ ЛОВИТЬ. For the last inequality, keep in mind that РЫБАЧИТЬ = ЛОВИТЬ РЫБУ. Since the universes of the lesser verb are not at all necessarily the universes of the larger verb, replacing the smaller verb in the sentence with a larger one also softens the violation of the axiom of the verbal cross. For example, the sentence “рыбаки ловят рыбу” no longer seems so strange, and even more so the sentence “рыбаки ловят судака”. At the same time, the sentences “рыбаки рыбачат” and “рыбаки ловят рыбу” are semantically equivalent.

Remark 5. There is a significant difference between the binary relation that is observed between objects and the correspondences given by the verbal cross. The binary relation S between the sets U and V is simply a set of pairs whose elements are connected by this binary relation. At the same time, the verbs * and + from Axiom A3 are mappings from the Boolean B_U to the Boolean B_V and vice versa. These are completely different mathematical objects. Based on the binary relation S, one can construct mappings from one boolean to another in various ways. The method used here is based on axioms B1 and B2 [1]. It turned out (Proposition 1) that the Booleans mappings constructed in this way satisfy Axiom A3, and thus the mappings Δ and ∇ are verbs.

Conversely, if Booleans mappings are given, then one can pass from them to the relation between universes in different ways. The mappings * and + satisfy axioms A1 and A3. This is tantamount to the fulfillment of Axiom A2 (Proposition 2) on the universes. Does this force you to choose a particular way of constructing a relationship from these mappings? Not at all. Moreover, these mappings are generally not bijections, in contrast to the mappings Δ and ∇. This can lead to differences in direct and reflexive verbs. For example, the mixing of two spaces in example 11 (Fig. 4*b*) [5] for the verbs ПОЕДАТЬ and ПОЕДАТЬСЯ generates a new category {Pe} in Fig. 4*c* [5], which leads to a violation of the bijection of the original mappings, since now the two categories {PPe} and {Pe} are mapped into one category {CRPe}. As a result, the isomorphism of spaces turns into an epimorphism, and the reflexive verb does not necessarily lead

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to the original category (i.e., in the general case, for the category X, instead of $X = X^{\Delta\nabla}$, we obtain only $X \subseteq X^{\Delta\nabla}$).

This is an important point for understanding the logic of verbs, which is discussed in [4]. When verbs in a sentence are connected by AND and OR operations, they form new mappings of new linguistic spaces (compound verbs). Axiom A3 still holds for these new mappings and, therefore, they are verbs. In this case, the dual isomorphism of spaces may give way to a dual epimorphism or monomorphism of the newly formed spaces. Thus, logical operations with verbs do not take mappings out of the verb zone, and this is an important point for the logic of natural language. At the same time, however, as we have seen, reflexive verbs for such compound verbs lose the return property to the original category. Perhaps that is why in the language we do not replace compound verbs with a new name of a new verb, but leave them in their original form connected by operations AND and OR of simple verbs.

To conclude this remark, we show the simplest way to move from the mappings * and + to the binary relation. To do this, it is sufficient to use mappings of singleton sets. Namely, the relation S_U is defined as the set of all pairs (x, y) such that $y \in \{x\}^*$. Similarly, the S_V relation is defined. It is not difficult to make sure that S_V is the inverse relation for S_U ($S_U = S_V^{-1}$). So, the two mappings actually generate a single relation $S=S_U$. The linguistic spaces constructed by this relation will be no larger than any other linguistic spaces constructed by the mappings * and +. In other words, categories from linguistic spaces constructed using S will also be included in any other linguistic spaces constructed using * and +. We will not prove this fact, since in the future only simple verbs like Δ and ∇ are used.

Despite the simplest mathematical background, in this subsection we are faced with a fundamental principle of linguistics that underlies the work of the mind: the principle of linguistic duality.

The principle of linguistic duality states that verbs and categories constitute a single whole, representing only a manifestation of different aspects of this unity.

In other words, if you agree that no collection of examples of a category allows you to go beyond this category, then you also acknowledge that the verbal cross of direct and reflexive verbs is performed for verbs, and vice versa.

This principle of linguistics is akin to the principle of duality in physics, which states that matter is a wave and a particle at the same time. Just as in physics, it is not difficult to show mathematically, but it is difficult to comprehend, and you just have to accept.

The principle of linguistic duality has many implications. For example, it means that teaching children a language can be conducted, both by pointing to examples of categories, and by pointing to examples of actions. Parents, of course, use both. The bias in one direction or another due to the principle of duality does not matter.

It also follows from the principle of duality: if categories are formed according to strict laws, then verbs in the language are completely involuntary. If some actions are present in the world around us, they are recognized by any consciousness in the same way as categories. And this also creates the basis for translation capability. On another planet, another consciousness will also select and form the verbs “eat”, “talk”, etc., if, of course, such actions are there.

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Remark 6. As an example of the interaction of categories and verbs, let us consider the nature of the occurrence of processes. Let, as a result of observing some interactions of objects, we have experimentally fixed a certain relation. There are no objects yet, and this relation consists of tuples of observable parameters. As a result of processing this relation, we have established that the multivalued dependence $X \twoheadrightarrow Y$ is fulfilled in it. This means that from all the observed data it is possible to isolate only the parameters included in XY and consider them independently of other measurements in the form of a binary relation S(X, Y). So, we have separated a part of the experimental data into a binary relation.

This binary relation can be used to construct space and co-space, connected by a direct and reflexive verb. At this stage, the formed verb is what holds (connects) the formed categories. The arity (valence) of a verb is determined by the number of categories that it holds together. Formed spaces are made up of categories, which in turn are made up of types. Types in this case are tuples of X or Y parameter values that behave exactly the same. In other words, types are generalized objects that make up categories. Types form disjoint sets on the universes. The types themselves are made up of many tuples that represent these generic objects. So, behavior forms concepts and generalized objects that are, as it were, indistinguishable from the point of view of this behavior (for this verb). Of course, if in addition to this we single out other binary relations, for example, the relation R(X, Z), then according to this relation another space and other types are formed on X (another partition). The intersection of two equivalences of types will split the universe into smaller classes (private objects), so that in the end, the types with relation to S(X, Y) will turn out to be composed of a certain number of private objects, and we will say that species consist of objects. For example, in relation to READING, people are grouped into certain categories (for example, fantasy lovers) and broken down into types. A type is a generalized reader. If we take into account other forms of human behavior, then this generalized view of the reader will be broken down into smaller subspecies until they are reduced to one person, after which we will begin to say that the types and categories of readers are composed of people.

So, behavior, through categorization, creates objects. Objects are still sets of tuples consisting of the values of the measured parameters. We can say that objects are boundaries within which objects are modified. These boundaries are determined by the identification algorithm by identifying tuples belonging to the same object. Now, if we observe some relation, then the change of tuples is interpreted by us as the “trajectories” of objects participating in the relation, within the boundaries of their types. These “trajectories” turn out to be similar whenever we observe a given relation, so that we have the opportunity to speak of the process as a trajectory of some type. Let's say the verb CIRCLE means cyclically changing values in tuples of objects. This would be impossible if the objects were not selected and the boundaries of identity (the glades where the actions of the objects unfold) were not defined. For example, for coordinate parameters, their observation ultimately leads to the concept of a trajectory of motion. A trajectory cannot exist without an object that moves along this trajectory. Even when we talk about a trajectory abstractly in a mathematical sense, even then we need an object – a point.

So, first behavior forms objects, and then objects form processes. In other words, first we view behavior as something that holds (connects) objects, and then we have the opportunity to look at the behavior from the point of view of what the objects “do” in the course of this behavior.

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These metamorphoses are ensured by the principle of duality through the unity and interdependence of categories and verbs.

In the language, the described transformations are reflected in the change in the meaning (semantics) of the verbs. For example, in the sentence “boy reads a book”, the verb READ acts as the name of the relationship that holds objects the BOY and BOOK together. On the other hand, in the sentence “it is very important to learn to read”, the verb READ already acts as a designation of a process that is characterized by certain “trajectories” of the actions of objects, by which we recognize the verb of reading when we observe people and books. If in the first case the valency of the verb READ is equal to two, then in the second it is zero. That is why, within the framework of the series, we did not use the concept of valence, but only arity.

The trajectories have their own characteristics and similar trajectories differ in the change of some of their characteristics. For example, children can CIRCLE at different speeds and this difference is expressed in the frequency of cyclical changes in tuples. We use adverbs to indicate the characteristics of trajectories. For example, we say: “The girl is circling slowly”, meaning the reduced frequency of cyclical changes in tuples. Compared to what is the girl's speed reduced? Relative to the reference frequency. But what is the source of this standard? Within the framework of this series, we have assumed that trajectory generators correspond to verb-processes in our minds. It is they that allow us to interpret the verbs of the language, to recognize the actions in the surrounding reality and to generate the actions of images in our consciousness.

So, adverbs (for the most part) are parameters of trajectories. They turn out to be possible only insofar as these trajectories are highlighted. In other words, the principle of linguistic duality generates processes, and through them – adverbs, so that we owe the appearance of adverbs in languages to the principle of linguistic duality.

These are, of course, not all the “effects” of the duality principle. For example, the emergence of the verbal metaphor is usually based on the similarity of trajectories, which is impossible without ideas about processes. For example, the verb “ХОДИТЬ” as a process looks like a certain trajectory of movement of the legs. The pendulum clock “ХОДИКИ” also “ХОДЯТ”, as the oscillations of the pendulum resemble the trajectory of the legs. This is where the metaphor “ВРЕМЯ ИДЕТ” appears, which originally meant the process of working in hours. The same can be said for the “ВРЕМЯ ТЕЧЕТ” metaphor, which seems to derive from the work of a water clock.

Conclusion. We examined the axiom for the verbs of the language (the axiom of the verbal cross) and found out that it is tightly connected with the axiom of categorization (the axiom of correctness). Although this connection is not identical, it becomes such with the additional use of the axiom of extensiveness. This relationship made it possible to formulate the fundamental principle of linguistics – the principle of duality. The manifestations of the principle of duality in the language are diverse, but in this article, we have limited ourselves to discussing the appearance of verbs-processes and adverbs in the language.

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No conflicts of interest related to this publication were reported.
Received 19.10.2022; adopted after review 23.11.2022; published online 24.04.2023.

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О конфликте интересов, связанном с данной публикацией, не сообщалось.
Поступила 19.10.2022; принята после рецензирования 23.11.2022; опубликована онлайн 24.04.2023.