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## Linguistic Data Model for Natural Languages and Artificial Intelligence. Part 7. Internal logic 2

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**Introduction.** The article continues a series of publications on the linguistics of relations (hereinafter referred to as R-linguistics) and is devoted to the study of the functioning of logical connectives with nouns and adjectives. The article is the second part of the discussion of internal logic, which examines the use of logical connectives within sentences. This research involves the formation of semantic logic, that is, logic that takes into account the semantics of sentences.

**Methodology and sources.** The results obtained in the previous parts of the series are used as research tools. To develop the necessary mathematical representations in the field of internal logic, the previously formulated semantic concepts and operations are used.

**Results and discussion.** Two types of negation are introduced and their properties are defined. The properties of the linguistic model are formulated. The use of logical connectives with nouns and adjectives is analyzed. It is shown that the connective NOT can be attributed to both external and internal logic. The connectives AND and OR do not have a logical content, but serve to identify the state of a particular concept. Three semantic substitutions are defined. The reasons for the stability of semantic interpretation when changing the state of relations are substantiated.

**Conclusion.** Abandoning the traditional view of natural language logic means abandoning logical operations and inference. This forces us to consider logical operations that now take into account semantics, since they are related to the structure of the linguistic model. Analysis of the functioning of logical connectives with nouns and adjectives shows that they either provide identification of the state of concepts, or can be transferred to external logic, or can operate within the framework of a linguistic model. The formulated substitution rules provide semantically correct substitutions and justify the stability of semantic interpretation in the conditions of changing states of concepts.

**Keywords:** R-linguistics, ascription operation, interpretation operator, substitution rule, semantics

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Оригинальная статья

## Лингвистическая модель данных для естественных языков и искусственного интеллекта. Часть 7. Внутренняя логика 2

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**Введение.** Статья продолжает серию публикаций по лингвистике отношений (далее R-лингвистика) и посвящена изучению функционирования логических связей с существительными и прилагательными. Она является второй частью обсуждения внутренней логики, которая изучает использование логических связей внутри предложений. Это исследование предполагает формирование семантической логики, т. е. логики, учитывающей семантику предложений.

**Методология и источники.** В качестве инструментов исследования используются результаты, полученные в предыдущих частях серии. Для разработки необходимых математических представлений в области внутренней логики применяются сформулированные ранее семантические понятия и операции.

**Результаты и обсуждение.** Введены два вида отрицания и определены их свойства. Сформулированы свойства лингвистической модели. Проанализировано использование логических связей с существительными и прилагательными. Показано, что связка НЕ может быть отнесена как к внешней логике, так и к внутренней. Связки И и ИЛИ не имеют логического содержания, а служат для идентификации состояния того или иного понятия. Определены три семантические подстановки. Обоснованы причины устойчивости семантической интерпретации при изменении состояния отношений.

**Закключение.** Отказ от традиционного взгляда на логику естественного языка означает отказ от логических операций и логического вывода. Это принуждает к рассмотрению логических операций, которые теперь учитывают семантику, поскольку связаны со структурой лингвистической модели. Анализ функционирования логических связей с существительными и прилагательными показывает, что они либо обеспечивают идентификацию состояния понятий, либо могут быть перенесены во внешнюю логику, либо могут действовать в рамках лингвистической модели. Сформулированные правила подстановки обеспечивают семантически корректные замены и обосновывают стабильность семантической интерпретации в условиях изменяющихся состояний понятий.

**Ключевые слова:** R-лингвистика, операция приписывания, оператор интерпретации, правило подстановки, семантика

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**Introduction.** This article continues a series of publications devoted to the introduction to the linguistics of relations (R-linguistics) – a formal direction in linguistics. Here we will continue the conversation about logic within the framework of R-linguistics representations.

In the previous part [1], the use of logical connectives with verbs was discussed within the framework of internal logic. It was shown that sometimes when using such connectives, it is

necessary to correct other members of the sentence, although in general logical connectives with verbs refer to external logic, that is, to logic in which the operands are sentences. In this part, we will study the use of logical connectives with other members of the sentence.

**Methodology and sources.** The results obtained in the previous parts of the series are used as research tools. To develop the necessary mathematical representations in the field of internal logic, the previously formulated semantic concepts and operations of the linguistic model are used.

*Logical operations with nouns and adjectives.* In [2], actions with categories and a dual isomorphism between spaces were defined. Let's add two more actions to the existing ones.

Let  $\mathbb{P}$  be a space,  $X, Y \in \mathbb{P}$ ,  $X \subseteq Y$ .

**Definition 1.** The category  $Z$  is called the intersection complement of  $Y$  for  $X$  (denoted by  $Z \in X \setminus Y$ ) if  $Y \cap Z = X$ . In general, the set  $X \setminus Y$  consists of more than one element.

It is also obvious that if  $X$  is an  $\cap$ -generator, then for any  $Y$  the only complement with respect to the intersection is  $X$ . Actually, this is the definition of an  $\cap$ -generator. It follows immediately from Definition 1 that  $X \in X \setminus Y$ , as well as the following

**Proposition 1.**  $Z \in X \setminus Y$  if and only if  $Y \in X \setminus Z$ .

**Proposition 2.** Let  $Z_1, \dots, Z_k$  be the complement by the intersection of  $Y$  for  $X$ , then  $Z = \bigcap_{i=1, \dots, k} Z_i$  is also the complement by about the intersection of  $Y$  for  $X$ .

*Proof.* By definition 1, we have  $X = (Y \cap Z_1) \cap \dots \cap (Y \cap Z_k) = Y \cap (Z_1 \cap \dots \cap Z_k) = Y \cap Z$ .

**Consequence.** According to Proposition 2, axiom B2 [2] holds in the set of categories  $X \setminus Y$ . If we choose the sum of all elements  $X \setminus Y$  as the universe, then we get the subspace of the original space, which consists of the complements by the intersection  $Y$  for  $X$ . Thus, the symbols  $X \setminus Y$  denote the subspace of the original space.

If  $\mathbb{P}'$  is a co-space for the space  $\mathbb{P}$  by the mapping  $\Delta$ , then by virtue of duality, the complement operation by addition  $X/Y$ , which is dual to the operation of complement by intersection, can be defined in the co-space.

**Definition 2.** Let  $X, Y \in \mathbb{P}$ ,  $Y \subseteq X$ . The category  $Z$  is called the complement of addition  $Y$  for  $X$  (denoted by  $Z \in X/Y$ ) if  $Y + Z = X$ . In general, the set  $X/Y$  consists of more than one element.

By virtue of the duality of space and co-space, for addition complements, the analogs of Propositions 1 and 2 are valid.

**Example 1.** Let's consider several examples of the introduced concepts, for which we use the relation shown in fig. 1a, and dual linguistic spaces constructed for it (fig. 1b), connected by a verb. In this fig., letters indicate the following objects and their properties:  $\mathcal{O}$  – equivalence,  $T$  – tolerance,  $\Pi$  – order,  $C\Pi$  – strict order,  $K$  – quasi-order,  $P$  – reflexivity,  $A_P$  – antireflexivity,  $C$  – symmetry,  $A_H$  – antisymmetric,  $A_C$  – asymmetric,  $T_P$  – transitivity.

Let, for example,  $Y = \{A_P, A_C, T_P\}$ ,  $X = \{T_P\}$  (fig. 1b on the right). Then  $X \setminus Y = \{\{P, C, T_P\}, \{P, A_H, T_P\}, \{P, T_P\}\}$ , and  $\{P, T_P\} = \{P, C, T_P\} \cap \{P, A_H, T_P\}$ . In addition, for example,  $\{P, C\} \setminus \{P, C, T_P\} = \{P, C\}$ , since  $\{P, C\}$  is  $\cap$ -generator. Similarly in fig. 1b to the left let  $X = \{\mathcal{O}, K, \Pi, C\Pi\}$  and  $Y = \{C\Pi\}$ , then  $X/Y = \{\{\mathcal{O}\}, \{\Pi\}, \{\mathcal{O}, K, \Pi\}\}$ , and all elements of the sets  $\{\{P, C, T_P\}, \{P, A_H, T_P\}, \{P, T_P\}\}$  and  $\{\{\mathcal{O}\}, \{\Pi\}, \{\mathcal{O}, K, \Pi\}\}$  are in one-to-one correspondence with respect to the mapping  $\Delta$ . In addition, for example,  $\{\mathcal{O}, T\} / \{\mathcal{O}\} = \{\mathcal{O}, T\}$ , since  $\{\mathcal{O}, T\}$  is  $\Sigma$ -generator.

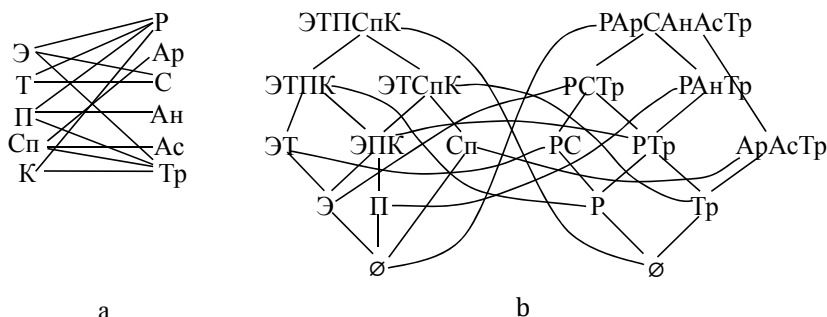


Fig. 1. Relation and its linguistic space

**Remark 1.** 1. The two introduced operations are actually ternary relations on a set of categories. In meaning, they vaguely resemble the operation of negation. Indeed, the operand  $X$  for the complement operation by addition specifies some universe or the largest element within which negation is considered. It must be said that the changing universe is not in logic, but it is in language. Consider the following example: “This year I will plant not an apple tree”. The category “not an apple tree” depends on the universe. Ask someone what I’m going to plant? Most likely, they will answer you that I will probably plant a pear, cherry or plum. In other words, the set “not an apple tree” is considered by us primarily within the framework of the closest category “fruit trees”, which acts here as a local universe. It is a very narrow universe; it does not even include birch, and even more so does not include shrub plants. If I persist: “No, not this,”– you will gradually expand the universe, and reach the bushes, and then, possibly, to vegetables. So, for the formation of NOT we always try to choose the closest categories from the narrowest universe, where the action of the verb is maximally similar. Also, this NOT is easier to define. Semantic logic changes universes like gloves, so the unary negation operation turns into a binary correspondence, one of the operands of which becomes the maximum element by addition or the minimum element by intersection.

2. By virtue of the duality of linguistic constructions (spaces – co-spaces), any way defined negation receives a dual analogue. Therefore, here too we get two types of negation. One negation (complement of intersection) includes those categories that provide separation from the original category ( $Y$ ). In this case,  $X$  acts as a “common place”. Category  $X$  is present in all categories of this scheme, and therefore it cannot be denied. This is “a yes and no” category. In this sense, this kind of negation is actually a negation of the  $Y-X$  elements within the allocated subspace. Moreover, already from Definition 1 it follows that always  $(Y-X) \cap (Z-X) = \emptyset$ . This ensures the fulfillment of the similarity of the law of contradiction for this type of negation ( $A \& \square A = A$ ). Similarly, for the complement of addition,  $X$  acts as a universe, and  $Y$  – as a negated category within the selected subspace. For this type of negation, due to duality, the law of exclusion of the third ( $A \vee \square A = I$ ) should be fulfilled. Unfortunately, this is not entirely true. The problem is related to the mismatch of  $\cup$  and  $\sum$ -generators. So, in fig. 1b on the left, the sum of two  $\sum$ -generators  $\{\Theta\}$  and  $\{\Pi\}$  generates an element  $\{\Theta\Pi\}$ , which has its own type  $\{K\}$ . For this reason, within the subspace  $\{\{\Theta\Pi\}, \{\Theta\}, \{\Pi\}, \emptyset\}$  the analogue of the law of exclusion of the third does not work. Indeed, the negation of the category  $\{\Theta\}$  in this space is the category  $\{\Pi\}$ . From the consideration of these two categories it does not at all follow that there are still some objects of the type  $\{K\}$ , which are not included in either  $\{\Theta\}$  or  $\{\Pi\}$  (although  $\{\Theta\} + \{\Pi\} = \{\Theta\Pi\}$ ). Theorem 5 [3] removes this problem for language spaces. Here again we are

faced with the fact when the language improves the logical properties of the noun-adjective part of semantic logic.

3. As noted, the negation of a category is a certain subspace of the original space. By virtue of Proposition 1 (and its dual sentence for another operation NOT), for any category  $Y$  for a given  $X$ , there is  $\text{NOT}(Y)$ , the negation of which is  $Y$ . Thus, the equality  $\square\square A = A$  is fulfilled here as one of the possible. The existence of several negations, ordered by inclusion in the form of a complete sublattice, allows us to speak of stronger, weaker, and incomparable negations. For example, for fig. 1b on the left for  $\{\text{ЭКПС}\}/\{\text{С}\}$  we obtain the following negation subspace:  $\{\{\text{ЭК}\}, \{\text{С}\}, \{\text{П}\}, \emptyset\}$ . Within this subspace, the negation  $\{\text{ЭК}\}$  is more complete than the negation  $\{\text{С}\}$  and  $\{\text{П}\}$ . In addition to the ranking of negatives, two more types of objects appear: “yes and no” (ID) and “neither yes nor no” (ND). For example, for  $\{\text{ЭТКП}\}/\{\text{ЭКП}\}$  there is only one negation –  $\{\text{ЭТ}\}$ . Here category  $\{\text{Э}\}$  is a category of type ID. Similarly, for the example  $\{\text{ЭКПС}\}/\{\text{С}\}$  and negation  $\{\text{С}\}$ , the category  $\{\text{П}\}$  is a category of the ND type.

It is easy to see that when passing to the Boolean as a space, when choosing  $U$  as the universe, and an empty set as the smallest element, both definitions of negation merge into one and turn into the classical negation we are used to.

4. Now let us formulate a set-theoretic model for noun-adjective logic within the framework of one (binary) relation.

On two sets  $U$  and  $V$ , called the universe and the co-universe, respectively, families of subsets  $\mathbf{P}U$  and  $\mathbf{P}V$  are given, called the linguistic space and the linguistic co-space, respectively. Moreover,  $U \in \mathbf{P}U$  and  $V \in \mathbf{P}V$ . Two bijections are defined on the families:  $\Delta: \mathbf{P}U \rightarrow \mathbf{P}V$  and  $\nabla: \mathbf{P}V \rightarrow \mathbf{P}U$  such that for any  $X \in \mathbf{P}U$  and  $Y \in \mathbf{P}V$   $(X\Delta)\nabla = X$ ,  $(Y\nabla)\Delta = Y$ . For any  $X_1, X_2$  from  $\mathbf{P}U$ ,  $Y_1, Y_2$  from  $\mathbf{P}V$ , the inclusion  $X_1 \subseteq X_2$  is fulfilled if and only if  $X_2\Delta \subseteq X_1\Delta$  (similarly for the inclusions  $Y_1 \subseteq Y_2$  and  $Y_2\nabla \subseteq Y_1\nabla$ ).

Four binary operations are defined on the families  $\mathbf{P}U$  and  $\mathbf{P}V$ .

1. For any  $X_1, X_2$  from  $\mathbf{P}U$ ,  $Y_1, Y_2$  from  $\mathbf{P}V$ ,  $X_1 \cap X_2 \in \mathbf{P}U$  and  $Y_1 \cap Y_2 \in \mathbf{P}V$  hold.
2. For any  $X_1, X_2$  from  $\mathbf{P}U$ ,  $Y_1, Y_2$  from  $\mathbf{P}V$ ,  $X_1 + X_2 = (X_1^\Delta \cap X_2^\Delta)^\nabla$  and  $Y_1 + Y_2 = (Y_1^\nabla \cap Y_2^\nabla)^\Delta$ .

The next two operations  $\backslash$  and  $/$  map two operands from  $\mathbf{P}U$  ( $\mathbf{P}V$ ) to the subspace  $\mathbf{P}U$  ( $\mathbf{P}V$ ).

3. For any  $X_1, X_2$  from  $\mathbf{P}U$  such that  $X_2 \subseteq X_1$ ,  $X_1/X_2$  consists of all  $X$  from  $\mathbf{P}U$  for which  $X_2 \cap X = X_1$ . The definition for the space  $\mathbf{P}V$  is similar.

4. For any  $X_1, X_2$  from  $\mathbf{P}U$  such that  $X_2 \subseteq X_1$ ,  $X_1/X_2$  consists of all  $X$  from  $\mathbf{P}U$  for which  $X_2 + X = X_1$ . The definition for the space  $\mathbf{P}V$  is similar.

5.  $\cup$  and  $\sum$  generators coincide in  $\mathbf{P}U$  ( $\mathbf{P}V$ ).

We will not list the properties of the operations. They follow from their definitions.

The model described above also has its own **substitution rule 2**.

*Let  $S$  be a verb,  $X, Y$  – categories and you can say the phrase  $XS$ . If  $X_1 \subseteq X$ , then we can say  $X_1S$  (similarly for  $Y$ ).*

Indeed, if  $X \times Y \subseteq S$  and  $X_1 \subseteq X$ , then  $X_1 \times Y \subseteq S$ .

Translated into the terms of signs [4], this can be reformulated as follows: if one category is characterized by all the values of signs of another category (plus some other signs values), then the first category can be substituted into the sentence instead of the second category. In fact, when, in reducing spaces, we expand the scope of the verb to the newly formed categories, the second



substitution rule is used. The same happened when discussing the OR operation with verbs [1]: taking into account the second substitution rule, we did not care at all about the categories in the co-space, because the semantic OR (mixing of spaces), introducing intersections, can only reduce the categories.

The second substitution rule appears at first glance to be the opposite of the syllogism of classical logic. The syllogism of classical logic in the set-theoretic interpretation looks like this: if  $A \subseteq B$  and  $x \in A$ , then  $x \in B$ . From the point of view of categories, the substitution rule 2 is the opposite: from the correctness of a phrase with a large category  $X$ , the correctness of a phrase with a smaller category  $X1$  follows. This feature is a consequence of the duality of spaces: a decrease in a category in space causes an increase in the corresponding category in co-space.

Consider a well-known example from Aristotle's logic: "All people are mortal". This means that the set of all people  $X$  has (the verb  $S$ ) a non-zero meaning of the sign "mortal". Socrates is a man. In other words, the set consisting of one element, Socrates, belongs to  $X$ . Following the second substitution rule, we obtain  $\{Socrates\} SY$ , that is, Socrates is also mortal.

The opposite situation (that is, replacing smaller categories in a sentence with large ones) takes place for sentences with fictitious indirect additions, which will be considered in a separate article, so for now the reader just needs to accept the validity of the following substitution rule on faith. Indirect additions correspond to multivalued dependencies, which allow decomposing relationships without loss of information. The property of a multivalued dependence to decompose a relation without losing information allows removing indirect additions associated with dependencies (fictitious indirect additions) without violating the correct expression. Consider the example of Bertrand Russell from the preface to [5] "Socrates was a wise Athenian". In accordance with the above, one can also say: "Socrates was wise", or "Socrates was an Athenian". So, the following **substitution rule 3** is valid.

Finally, we present one more **substitution rule 4**, which links the internal and external semantics of sentences. We'll start with a simplified case.

*If  $S$  is a verb and  $X, Y$  are categories,  $Y \cap (notY) = \emptyset$  and you can say the phrase  $XS(notY)$ , then you can say  $X(notS)Y$  (similarly for  $X$ ).*

Finally, we present one more substitution rule 4, which links the internal and external semantics of sentences. We'll start with a simplified case.

*Если  $S$  – глагол и  $X, Y$  – категории,  $Y \cap (неY) = \emptyset$  и можно сказать фразу  $XS(неY)$ , тогда можно сказать  $X(неS)Y$  (аналогично для  $X$ ).*

In accordance with Definition 1, here the complement by the intersection for the empty set (in Definition 1  $X = \emptyset$ ) is used as "not". Hence, it is clear that in relation to  $XS(notY)$  there will not be a single element from  $XS$ , whence it follows that  $X(notS)Y$  is true, where "not" with a verb means "not" from external logic [6]. For the same reasons, the substitution is valid for  $X$ .

For example, from the correctness of the phrase: "Farmers do plant not apple trees," the correctness of the phrase follows: "Farmers do not plant apple trees". By virtue of the equivalence of the two categories of attitudes from the correctness of the phrase: "Not farmers plant apple trees" – the correctness of the phrase "Farmers do not plant apple trees" also follows.

As we can see from the rationale for this substitution rule, its semantics are, as it were, divided into two parts. One part states that pairs from  $XS$  should not be included in the semantics of the

text, which means that, in fact, the text contains a sentence  $X(\text{not}S)Y$ . This fact just confirms the simplified substitution rule 4. On the other hand, the semantics of the text can include pairs from  $XSV$ , where  $V$  is a category for the verb  $S$  such that  $Y \cap V = \emptyset$ . Let's consider this second part of semantics in more detail.

It follows from Proposition 2 that Axiom B2 holds for complements by intersection. To define a subspace, it is enough for us to “correctly” choose the universe in order to determine the B1 axiom. For example, in fig. 1b, when using the complement by intersection, the subspace consisting of the elements of  $\text{ЭТКП}$ ,  $\text{ЭКП}$ ,  $\text{ЭТ}$ ,  $\text{П}$ ,  $\text{Э}$  and  $\emptyset$  is used as “notCп”. Any element of this subspace can act as a category  $V$ , if  $Y \cap V = \emptyset$ . Thus, the category  $V$  is actually a variable whose domain of definition is the subspace of all intersection complements. This largest subspace can be very significant and immeasurable in the process of communication. Therefore, people narrow the scope of the variable  $V$  by choosing the closest category containing the denied category as the area where negation is considered. So, in fig. 1b, the nearest larger category for Cп is  $\text{ЭПКП}$ . The choice of this category as the area within which the negation of “notCп” is considered, specifies the subspace of the complements by the intersection, consisting of the  $\text{ЭКП}$ ,  $\text{П}$ ,  $\text{Э}$ , and  $\emptyset$ .

Let's go back to the farmer's example. In this example, we will first assume that the subspace with the fruit tree universe is chosen as the subspace where negation is considered. If I persistently deny your assumptions, then at some point you will have to expand the universe within which denial is considered. For example, for fig. 1b, it will be necessary to switch from the  $\text{ЭКП}$ Cп universe to the  $\text{ЭТКП}$ Cп universe, which will allow increasing the subspace of complements by intersection. In particular, now it will include the category  $\text{ЭТ}$ , which does not have any properties in common with the category Cп. Notice how the range of the variable “something” changes when you move from the sentence “farmers are planting something” to the sentence “Farmers are planting something, but not apple trees”. So, the scope of definition of the variable  $V$  expands only in conditions of extreme necessity.

So, the phrase “Farmers do not plant apple trees” is equivalent to the semantics of two phrases: “Farmers do not plant apple trees” and “Farmers plant some other fruit trees”. Here, the role of the variable is played by the phrase “some others”, and the farmers in the two sentences are the same people. So, a sentence with a negation of a noun is semantically equivalent to two sentences: a negative sentence and a sentence with a variable.

A denied category can have more than one closest category. For example, the  $\text{ЭКП}$  category in fig. 1b has two closest inclusive categories:  $\text{ЭКП}$ Cп and  $\text{ЭТКП}$ . This suggests that we can consider denial in terms of various types of similarities. The choice of  $\text{ЭКП}$ Cп allows us to consider negation in the framework of transitive relations. In this case, the intersection complement subspace will contain the elements Cп and  $\emptyset$ . The choice of  $\text{ЭТКП}$  allows us to consider denial in the framework of reflexive relations. In this case, the intersection complement subspace consists only of the empty set. Perhaps, when the denied category has more than one closest one, people choose the sum of these closest categories as the universe on which the intersection complement is considered.

In the linguistic space in which negation is considered, the smallest element can also be a non-empty set  $Z$ . In this case, a special reference is made to this set in the sentence, since it corresponds to the category of type ID (“both yes and no”), since  $Y \cap (\text{not}Y) = Z$ . This indication usually has

the expression “except Z”, which is appended to the left and right side of the substitution rule. Finally, the category closest to the negated category can be a U-generator with its own type. For example, in fig. 1b (notII) has the negation of E and  $\emptyset$  within the framework of the nearest category of ЭКП, which is an U-generator with its own type K. Element K in this construction is an element of the type ND (neither yes nor no), which leads to violation of the law of exclusion of the third for reasons other than Russell's famous example (“the current king of France is bald”).

So, the appearance in the text of a sentence with the negation of a category semantically means the appearance of a negative sentence, which prohibits the interaction of two categories (noun and object) and an affirmative sentence with a variable, the scope of which may change depending on the context.

Let's return to the previously described model of two interconnected spaces with the operations specified on them. It would seem that the above construction should cause some bewilderment. Indeed, in the language there are many verbs between different universes, and according to this the model should not be so simplified, containing only one verb and one relation. This is all true. But, as we saw in [1], problems of plurality of verbs are solved using logical connectives with verbs, which can influence the form of nouns. It would seem that we now need to consider other logical connectives with nouns, since they are widely used in the language. So far we have only considered the interaction of nouns with “NOT”. However, as will be shown later, logical connectives with nouns actually perform a different function, so there is no need for further consideration at this time.

After there has been a transition from the representation of categories as a set of objects, to categories, in the form of identification algorithms (conjunctions of signs values), actions in linguistic spaces are described in terms of adjectives, so that all logic in these spaces is determined by the systematization relationship between objects and their signs, and the model defined in this section is, in fact, a model for a systematization relation, that is, the relation between objects and signs values, for which all the substitution rules described above are valid.

Although the systematization relation is often an ordinary binary relation, there is a feature that allows you to identify it in the language. Consider the phrase: “Этот парень не орёл”. Applying the fourth substitution rule, we get: “Этот парень не есть орёл”. We can see that nothing has changed with the transfer of “NOT”. These phrases are identical even without an additional variable clause. What happened? We are not looking at the behavior of objects, but the relation of having the values of signs, because “being an eagle” is a sign of a male person, and not some action. In this sense, the verb “is” is not formally a verb. That is why in the initial version of the phrase we did not even mention it. The verb “is” is a technological technique that allows you to unify the grammar of a language by reducing this type of utterance to a typical binary version. Its existence is a technological invention of our ancestors and not only ours.

It follows from the definition of the sign that all objects of the universe have a zero value of the sign. This was due to the need to combine in one relation two relations: the relation of possession of a feature and the relationship of possession of the meaning of a feature. In reality, if we determine the value of a sign for some object, then if the object has a nonzero value of this sign, we will always determine only this value as a result. If the object does not have this sign, we will always get a zero value. Thus, for simple signs with one non-zero value, the negation of the



sign value means that objects with this sign value will no longer have other non-zero values and, therefore, will not have pairs with the verb “is” so the category will not have this sign.

As was shown in [4], multivalued signs are obtained from simple ones based on the “non-intersection” relation of the corresponding  $\cap$ -generators. In general, there may be several such variants of multi-valued features, and when considering logical operations, we do not know which option was chosen (which multi-valued features were formed). Therefore, semantically, when using the “NOT” operation, all signs are still assumed to be simple. Consider the sentence “the relation is not symmetric”. It means that the “relation” object does not have the sign of symmetric. Therefore, the transfer of negation to the verb is identical to the original sentence. At the same time, above we used a multi-valued scale of the SYMMETRY sign with non-zero values “symmetric”, “antisymmetric”, “asymmetric”, but with the operations “NOT” we automatically assume the SYMMETRY sign to be simple, just as we assume the signs ANTISYMMETRIC and ASYMMETRY to be simple.

So, for the systematization of object-sign relations, the substitution rule turns into an identity without an additional sentence with a variable, which is a litmus test for identifying the sign. Thus, for the systematization relation, the transfer of negation from a sign to a verb does not give anything, which means that the question of the negation of signs in the systematization relation must be resolved without taking into account verbs; only in the interaction of nouns and signs. This, of course, does not apply to “not” with nouns that do not play the role of signs. The phrase “not this guy is an eagle” to go to the identity requires an additional sentence with the variable: “This guy is not an eagle, but some guy is an eagle”. From all that has been said, it follows that the set-theoretic model formulated in this article does not concern behavioral verbs, but relations of concepts and signs.

As an example, we can cite the work [7], where in paragraph 2 of Chapter 6 the transformation of language judgments, called transformation, is considered. In particular, the authors write: “The transformation of judgments is a logical device by which an affirmative judgment is transformed into a negative one or a negative one into an affirmative one, but the meaning of the judgment does not change”. By this they mean that if the complement and the verb have negations, then this judgment is equivalent to the same judgment, but without negations. According to the fourth rule of substitution, it is really possible to transfer the negation from the object to the verb, but it should be remembered [6] that the rule for double negation of the verb in the general case is not fulfilled in the language, which the authors did not take into account. As we have seen, the reverse can be done only in the case when it comes to the relation “objects-signs”. Therefore, in general, in this work, this transformation is interpreted erroneously. However, apparently feeling this, the authors add at the end of the paragraph: “Wrong transformations (?) lead to uncertainty in judgments or to a direct distortion of their meaning”.

Finally, let's look at some examples of logical operations on the model defined above. To do this, we will use the linguistic relationship space (fig. 1b) as a space, and the recognition tree shown in fig. 2 as a co-space.

This tree is actually an inverted co-space. It easily turns into a co-space if we unite the vertices marked with the same relations symbols:  $Tp(K)$  with  $P(K)$ ,  $C(\Theta)$  with  $Tp(\Theta)$ ,  $A_H(P)$  with  $A_H(P)$ . For convenience, in this tree, object and feature designations are combined, namely, objects from their own types corresponding to these vertices are assigned to the values of signs in the vertices

in brackets. The arcs are marked with the analyzed signs, and the vertices are marked with their values. Consider, for example, complement by addition  $(\exists \text{TKII})/(\exists \text{T})$ .

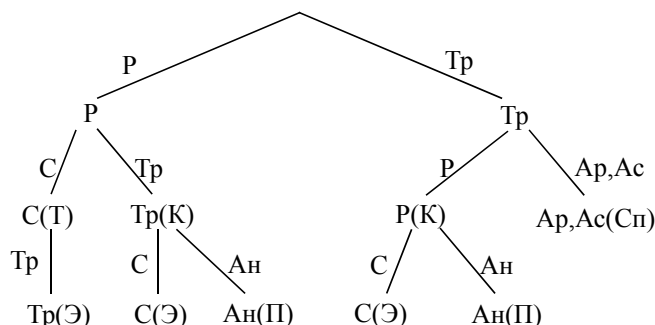


Fig. 2. Co-space in the form of a tree of recognition

In the recognition tree, these operands correspond to vertex P (reflexive relations) and vertex C(T). To obtain additions for summation, it is necessary to consider a subtree with vertex P. One (left) branch leaving this vertex contains the denied category C(T), and the categories located on the right branch just give the vertex P. However, the category  $\exists$  enters both the vertex C(Э) and the vertex Tp(Э), which is located below C(T) and therefore, when added to C(T), gives C(T). Thus, only two possible vertices remain: Tp(K) and AH(P). So, for reflexive relations, the negation of symmetric relations is transitive relations and, in particular, transitive and antisymmetric relations. This can be written in the form  $(\exists \text{TKII})/(\exists \text{T}) \cong (\exists \text{KII})$  and  $(\exists \text{TKII})/(\exists \text{T}) \cong (\text{II})$  ( $\cong$  reads as “including equals”). In fact, we have used the duality of the two types of negation, since we have reduced the search for complements by addition to the search for complements by intersection in the co-space of features. So, if  $X = \{P\}$ ,  $Y = \{P, C\}$ , then Z is equal to either  $\{P, \text{Tp}\}$  or  $\{P, \text{Tp}, \text{AH}\}$ . Similarly  $(\exists) \setminus (\exists \text{T}) \cong (\exists \text{KII Cn})$  and  $(\exists) \setminus (\exists \text{T}) \cong (\exists \text{KII})$ .

Generally speaking, by virtue of duality, it does not matter whether we returned to the object space as a result of applying operations or remained in the signs co-space, since they are one and the same. That is why the model defined above determines the semantic logic of the interaction of nouns and adjectives, in contrast to the model of classical logic, where there is only one universe with operations of negation and intersection. In addition, the fact of the presence of a co-space of signs creates some additional possibilities. Let's say  $(\exists \text{T})/(\exists) = (\exists \text{T})$ . In this case  $(\exists)$  is a category of the ID type, and in this sense, it has no negation. However, in the tree fig. 2 one branch goes from the vertex C(T) to the vertex Tp(Э) with a check of the sign Tp. Denying the meaning of this sign leaves us in the universe  $(\exists \text{T})$ , but does not lead to the category  $(\exists)$ . In other words, to the question: “What is inequivalence within the framework of tolerances?”, – one can answer: “Non-transitive tolerances are nonequivalence”. This is a mathematically meaningful phrase. Such negation cannot arise in classical object space, because in this example “NOT” separates not categories, but species. This kind of negation can exist only when using adjectives (signs), when “NOT” is transferred from categories to signs or parameters and it is not necessary to return back to object space. In other words, internal logic operates on the joint field of nouns and adjectives or on the model of two dual spaces.

We have the same in understanding the plurality of negations. In the presence of adjectives (signs and parameters), the negation of the category leads to the question: “In what sense is this

negation considered?” Let's say for (ЭКПСп)/(Сп) we have three values: (ЭКП), (П) and (Э). We are looking for negation within a transitive relationship for a relationship with properties Ap and Ac. If we deny the antireflexivity of the category (Сп), then we get transitive and non-antireflexive relations. All reflexive relations are present in the universe as non-anti-reflective relations. Since we only consider transitive relationships, tolerance falls out of this list. Remains (ЭКП). You can also consider negation by the parameter of asymmetry. Asymmetry has two negative meanings: “symmetric” and “antisymmetric”. Among transitive relations, they correspond to equivalence and order. So, the negation of a strict order has two meanings: its negation as an antireflexive relation and negation as an asymmetric relation, which exactly correspond to the categories (ЭКП), (Э) and (П). In language this is expressed by the transfer of “not” to this or that adjective. From this point of view, just a single negation for the language would seem very strange. Indeed, in the conjunction of signs that identify a category, negation according to de Morgan's rules means negation of the values of signs connected by the OR operation. Thus, in the general model of two spaces, various variants of category negation arise.

**Results and discussion.** So, we have considered various options for the functioning of logical connectives in the language. Even when these bundles relate to the interaction of sentences, as was the case with verbs, it is often necessary to adjust the categories. This becomes even more noticeable when nouns and adjectives interact. For this interaction, the transition to connections between sentences is no longer possible, so that they cannot be interpreted within the framework of external logic. For example, for categories, we considered two options for negating them. One option results in a negative sentence and an additional affirmative variable sentence. In fact, this variant of external logic (since negation carries over to the sentence predicate) that reformats the text somewhat. The second option is associated with the interaction of nouns and adjectives within the internal logic of sentences.

*Condition problem.* People in their activities come across only examples of categories, often very limited and different for different people. For this reason, it might seem that different people should have different category systems. Why are language categories so stable and recognized by all native speakers?

If you look at categories as a set of examples, then the current filling of categories with examples is different for different people. The phrase: “Farmers do plant not apple trees” – for representatives of different regions will have different content. Representatives of the northwest will think that farmers are probably planting pears, cherries or plums, while representatives of the south of Russia may well have a different assessment. We are talking not only about individual objects, but also about the current content of the categories. For example, the category “fruit trees” includes trees from which we eat and which we tend to plant on our plots. For the St. Petersburg region, this category includes apple, pear, plum and cherry trees. But for the Krasnodar region, it also includes other trees, for example, merry, apricots, etc.

So, the real set of examples of a category that is reported in a proposal may differ significantly from the set of examples that make up a category. This raises two problems. The first problem is to substantiate the effectiveness of logical-semantic conclusions in the case of changes in the observed states of the relationship. The second problem has to do with the need to somehow identify the state of the relationship when it matters. That is, it is necessary to indicate the state of

the relationship in question. Let us first discuss the first problem, when categories are considered as sets of examples.

Let  $S$  be a binary relation on  $U \times V$  and  $A \subseteq U$  ( $B \subseteq V$ ). The restriction of a binary relation  $S$  to  $A$  ( $B$ ) is a binary relation  $S'$  such that  $(x, y) \in S'$  if and only if  $(x, y) \in S$  and  $x \in A$  ( $y \in B$ ).

**Proposition 3.** If  $U^\Delta \neq \emptyset$  ( $V^\nabla \neq \emptyset$ ), then for any  $\cap$ -generator there is an element  $y$  ( $x$ ) such that  $y^\nabla$  ( $x^\Delta$ ) coincides with this generator. If  $U^\Delta = \emptyset$  ( $V^\nabla = \emptyset$ ), then the proposition is valid for all  $\cap$ -generators, excluding  $U$  ( $V$ ).

*Proof.* Let  $X$  be an  $\cap$ -generator for which  $X^\Delta = Y \neq \emptyset$ . Then  $X = \bigcap_{y \in Y} y^\nabla$ . If all elements in  $Y$  are such that  $X \subset y^\nabla$  and their intersection is equal to  $X$ , then  $X$  is not a  $\cap$ -generator. Consequently,  $Y$  has no such elements and  $X$  coincides with some category  $y^\nabla$  ( $y \in Y$ ).

The converse is, of course, not true. An arbitrary category  $y^\nabla$  is not necessarily a  $\cap$ -generator. For example, in fig. 1b  $K^\Delta = \{P, Tp\}$ , but  $\{P, Tp\}$  is not  $\cap$ -generator in the right co-space.

**Proposition 4.** Let  $S$  be a binary relation and  $S'$  be its restriction to  $A$ . Then the co-space of the relation  $S'$  belongs to the co-space of the relation  $S$ , and the space of the relation  $S'$  consists of categories that are the result of the intersection of the set  $A$  with the categories of the relation space  $S$ .

*Proof.* In accordance with Proposition 3, any  $\cap$ -generator of a co-space with respect to  $S'$  is a set  $x^{\Delta'}$  ( $x \in A$ ). Therefore, the set  $\cap$ -generators of the co-space with respect to  $S'$  is a part of  $\cap$ -generators of the co-space with respect to  $S$ , and hence the co-space with respect to  $S'$  belongs to the co-space with respect to  $S$ . Similarly, any  $\cap$ -generator of the space by  $S'$  is a set of the form  $y^{\nabla'} = y^\nabla \cap A$ . Since all categories of the space with respect to  $S$  are the intersection of some  $\cap$ -generators, all categories of the space with respect to  $S'$  are obtained from the corresponding categories of the space with respect to  $S$  by their intersection with the set  $A$ .

From sentence 4 it follows that the relation  $S'$  forms a smaller verb on the co-space than the verb formed by  $S$ . In accordance with the verb substitution rule, the use of the “old” (larger) verb in the sentence is legitimate and even equivalent (since in the sentence for the larger verb uses the same categories as the smaller one). So, with a lack of information related to the narrowing of one universe, you can use the old verbs in sentences without collisions on another universe. If we now turn to the contracted universe, then since  $X \cap A \subseteq X$ , then in accordance with the second rule of substitution, the correctness of the application of the original verb is preserved even when it is transferred to the contracted category  $X \cap A$ . So, the reduction of categories as a result of narrowing the observed objects does not change the correctness of the application of the original sentences. Of course, when the narrowing occurs at two universes, the above reasoning will have to be done twice. However, it should be borne in mind that with the restrictions for the remaining elements of the universe, all existing connections must remain. For example, if we consider the READING relation for readers of a district, and then narrow it down to readers and books of one library, then all pairs (READER, BOOK) from the library cards should be retained.

Let us now turn to the case where we look at categories as identification algorithms. In accordance with the definition of mutual dependence [4], the narrowing of the universe of objects to the set  $A$  preserves dependence in a new relation. This follows directly from the definition of mutual dependence. It is also important here that the remaining objects retain their relationship links. Since multivalued and functional dependencies are a special case of mutual dependence, this property is also valid for them. For convenience, we will consider only simple signs (atomic elements of the

systematization lattice). Preservation of dependencies means that, for categories, the identification algorithm still looks like a conjunction of checks on the values of simple signs (Lemma 15 [4]).

In accordance with Proposition 4, the narrowing of the set of objects of the relation OBJECT-SIGNS may lead to the disappearance of some of the sign values. Since simple signs have only two values, and a zero value cannot disappear (by definition of a sign), only a single value can disappear, which indicates that the object has this sign. In other words, as a result of narrowing, objects with some sign may simply not get into it. A feature with one zero value, by definition, corresponds to a new universe (narrowing of objects) and in the conjunction of the analysis of feature values will always be true. Thus, the “zeroing” of some features will not change the result of conjunctive analysis in any way for all objects remaining upon narrowing. In other words, the “old” identification algorithm will still work flawlessly, and the reference to it, as an object representing a category, retains its validity. We can say that the stability of categories in a language with the widespread instability of their functioning has the same nature as the stability of dependencies in relational databases, which determines the invariability of the database schema when the state of relations changes (changing filling of relations with tuples).

Since the data dependencies are not affected by specific states of relations (observed data), the signs also do not depend on the states of the relationship, which means that they define a stable system of categories by their values. That is why, in the law of creative thinking [8], after the identification of signs, a change of causes and effects occurs: the reason for the entry of an object into a category is no longer its behavior, but the meaning of its signs.

After the recognition system has been created, the objects of the categories are characterized by signs and the volume of the category is determined by the set of all examples that have the necessary signs values. Some features that are not included in this analysis may differ, but this does not matter for the system of assigning objects to a category. From this point of view, all objects that have the necessary signs values are the same for the recognition system. As a result, the recognition system sets the maximum category size. Therefore, if we observe only some of the examples of a category, then all verbs and all substitution rules apply to it.

**So, changing the volume of a category while maintaining the values of signs for objects of this volume does not change the semantic reliability of the original phrases containing the name of the category.**

The second problem associated with the change in the volume of categories gives rise to the need to sometimes indicate what kind of content of the category is being discussed. “Schoolchildren have come to our excursion”. Of course, these are not all schoolchildren in the world, but only some specific students. If in the future in our speech we again need to use the category “schoolchildren”, then it will be necessary to somehow indicate that we are talking about the same composition of the category “schoolchildren”. This problem is solved in the language by some logical actions, although, strictly speaking, they are not related to logic.

Consider as an example the phrase: “In this place I want to plant an apple or pear tree”. We know that the categories “apple” and “pear” are combined into the category “fruit trees”. But the speaker does not mean the phrase: “In this place I want to plant a fruit tree”. If the speaker had meant this, he would have said so directly, or, for example, said: “In this place I want to plant something like an apple or pear tree”, thereby creating some variable with the definition area “fruit



trees”. Connecting a logical connective of categories from a more general category is an indication of a different origin of the logical connective. In fact, there are two sentences: “In this place I want to plant an apple tree” and “In this place I want to plant a pear”. These two sentences are connected by a connective OR, and the example actually refers to external logic, although the connective OR is transferred inside the sentence. The pronoun “I” is a language variable that accepts values from people universe. In order to show that we are talking about the same meaning of “I”, the connective OR is introduced inside the sentence, combining additions, but it is not an operation OR (summation operation) in the linguistic space.

Let's consider another example: “Teachers and students took part in the exhibition”. What does the union AND mean in this sentence? Of course, we are not talking about the intersection of the categories STUDENTS and TEACHERS in the linguistic space. Nor is it a general category covering teachers and students. Here again we are faced with two sentences, which are combined into one because of the need to show the identity of exhibitions in these sentences. And again, this is achieved by transferring the operation of joining sentences from the external logic inside the sentence. And again, the connective does not mean action in the linguistic space on the PEOPLE universe.

**Conclusion.** Let's summarize the analysis of the use of logical connectives in the language. Logical connections with verbs are directly operations of external logic. Even when we are talking about the same universes of concepts participating in a sentence, even then these operations remain operations of external logic with the addition of some noun transformations. The use of logical connectives OR and AND with nouns is usually associated with the need to indicate the identity of the state of the concepts used in the sentence. These operations do not mean the formation of new concepts and (taking into account the identity) can also be brought into external logic. Even the operation NOT with nouns can be transferred into external logic by transferring NOT to a verb and with the possible addition of a sentence with a variable. On the other hand, the operation NOT with nouns can be implemented in the model of connected spaces of object-signs, which refers to the internal logic of the sentence. From a consideration of this model, it is easy to understand that logical operations with definitions are also implemented within a sentence on the specified model. So, outside of the external logic, with a few reservations, only the interaction of objects-signs appears.

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